

Parity violation through color superconductivity

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Abstract

We give a pedagogical discussion of how color superconductivity can produce parity violation in cold quark matter at very high densities.

In this note, we give a pedagogical discussion of how, for massless quarks at very high densities, the formation of a spin-zero color superconducting condensate spontaneously breaks both the axial $U(1)$ symmetry and parity [1]. This observation is implicit in the seminal work of Bailin and Love, is noted by Alford, Rajagopal, and Wilczek, and is explicitly discussed by Evans, Hsu, and Schwetz [2].

For simplicity, consider two degenerate flavors of quarks, and assume that a quark-quark condensate forms in the color-antitriplet channel [1, 2, 3, 4]. For massless quarks, two of the four possible condensates with total spin $J = 0$ are [1]

$$\langle \phi_1^a \rangle = \epsilon^{abc} \epsilon_{fg} \langle q_f^{bT} C \gamma_5 q_g^c \rangle \quad \text{and} \quad \langle \phi_2^a \rangle = \epsilon^{abc} \epsilon_{fg} \langle q_f^{bT} C \mathbf{1} q_g^c \rangle, \quad (1)$$

where $a, b, c = 1, 2, 3$ are $SU(3)_c$ color indices, $f, g = 1, 2$ are $SU(2)_f$ flavor indices, and C is the charge conjugation matrix. $\phi_{1,2}^a$ are antitriplets under $SU(3)_c$ gauge transformations and singlets under $SU(2)_f$ rotations [1]. The condensate ϕ_1^a is even under parity, $J^P = 0^+$, while ϕ_2^a is odd, $J^P = 0^-$. There are two other condensates [1], but they do not change our qualitative arguments about parity violation, and so we omit them.

In the limit where mass and instanton-induced terms can be neglected, the effective Lagrangian for color superconductivity is

$$\mathcal{L}_0 = |\partial_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 + \lambda \left(|\phi_1|^2 + |\phi_2|^2 - |v|^2 \right)^2, \quad (2)$$

where $|\phi|^2 \equiv \sum_a (\phi^a)^* \phi^a$. When mass and instanton effects are neglected, the Lagrangian is symmetric under axial $U(1)$ transformations, which rotate ϕ_1^a and ϕ_2^a into each other. Therefore, there is only one quartic coupling, λ . The Lagrangian (2) generates nonzero vacuum expectation values for the ϕ^a 's, which can be written as

$$\langle \phi_1^a \rangle = v^a \cos \theta, \quad \langle \phi_2^a \rangle = v^a \sin \theta. \quad (3)$$

Condensation picks out a given direction in color space for v^a , and a given value for θ . $v^a \neq 0$ breaks the $SU(3)_c$ color symmetry, which produces color superconductivity. $\theta \neq 0$ breaks the axial $U(1)$ symmetry. Further, whenever $\theta \neq 0$, there is a nonzero $J^P = 0^-$ condensate $\langle \phi_2^a \rangle$; this represents the spontaneous breaking of parity (relative to the external vacuum).

This breaking of parity is actually familiar from the spontaneous breaking of chiral symmetry. Consider two flavors of *massless* quarks; the effective potential is $O(4)$ -symmetric, involving the $J^P = 0^+$ σ - and $J^P = 0^-$ π -meson fields. For massless quarks, it is as likely for a parity-odd pion condensate to form as it is for a parity-even σ -meson condensate. This does not happen in nature, because nonzero quark masses break chiral symmetry explicitly, and thus favor a 0^+ condensate.

Similarly, it is important to add to the effective Lagrangian (2) terms which explicitly break the axial $U(1)$ symmetry:

$$\mathcal{L}' = -c \left(|\phi_1|^2 - |\phi_2|^2 \right) + m^2 |\phi_2|^2. \quad (4)$$

As shown by Berges and Rajagopal [4], the first term is due to instantons, with c proportional to the instanton density. Instantons are attractive in the $J^P = 0^+$ channel, and repulsive in the $J^P = 0^-$ channel, so c is positive.

In the second term, each power of the current quark mass m_q is accompanied by one power of ϕ_2^a . Since ϕ_2^a itself is not gauge invariant, the simplest gauge-invariant term is $m_q^2|\phi_2|^2$ [1], so $m \sim m_q$. Thus, the pseudo-Goldstone boson for the axial $U(1)$ symmetry is extremely light, $m \sim 10$ MeV, taking m_q to be the up or down quark mass and assuming the constant of proportionality between m and m_q to be of order 1. This is in contrast to the explicit breaking of chiral symmetry, where the corresponding term is linear in the quark mass. The pseudo-Goldstone bosons are the pions which are relatively heavy, $m_\pi \simeq 140$ MeV $\sim \sqrt{m_q}$.

Both instanton and mass terms act to favor the formation of the 0^+ condensate ϕ_1 over that of the 0^- condensate ϕ_2 . Consider, however, the limit of very high densities. When the quark chemical potential $\mu \rightarrow \infty$, the instanton density and so c vanish like $\sim \mu^{-29/3}$ (for two flavors). The real question is whether at some density the current quark mass is negligible compared to the scale of the condensate. If this happens, we reach an “instanton-free” region in which quarks are effectively massless, \mathcal{L}' can be neglected, and parity is spontaneously broken.

Because mass terms are always present, the true thermodynamic ground state is always the parity-even 0^+ condensate, i.e., $\theta = 0$. There is, however, a finite probability for the system to condense in a parity-odd state, i.e., $\theta \neq 0$. The size and lifetime of this state is set by the mass of the pseudo-Goldstone bosons. For chiral symmetry breaking, the characteristic scale is $1/m_\pi \sim 1.4$ fm. This is small compared to the time and length scales of a heavy-ion collision, so that parity-odd fluctuations average to zero. On the other hand, the region in space-time over which a parity-odd color superconducting condensate forms is large, $1/m \sim 20$ fm. If the collision time is shorter than this time scale, there is a finite probability that the system decays in a parity-odd state. We therefore propose to trigger on phase-space regions where nuclear matter is cool and dense, in order to observe the formation of parity-odd color-superconducting condensates on an event-by-event basis. A possible global parity-odd observable was discussed in [5].

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